2.2 PROPAGATION PROCESSES (Robert Qiu)

2.2.1 INTRODUCTION

The theoretical and mathematical foundation of UWB signal processing and system modeling is based on the interaction of a transmitted pulse with the transmission medium, e.g., free space, buildings, furniture, trees, hills, rivers, stones, etc. The historical logical development of the UWB propagation processes can be conveniently divided into four stages: (1) fundamental concepts and mechanisms; (2) high-frequency approximation techniques (GTD/UTD); (3) transient and impulse response approximations; (4) per-path pulse waveform distortion. First the physical cornerstone to understand the medium-interaction mechanisms of UWB pulse propagation and shape distortion was the work of Sommerfeld (1901) on the diffraction of pulse by a perfectly conducting half-plane [1]. His solution was the first exact solution of Maxwell’s equations. Lamb (1910) [2] and Friedlander (1946) [3] already studied the pulse distortion caused by diffraction at a half-plane. These solutions provided us good pictures of understanding the physical mechanisms of diffracted pulses [4-7]. These solutions were impractical in engineering and applied science. Secondly another conceptual breakthrough was the birth of the Geometric Theory of Diffraction (GTD) developed by J. B. Keller (1962) [8], based on the Lunberg-Kline series in 1949 [7]. Rather than seeking the exact solution for Maxwell’s equations, GTD uses a generalization of Fermat’s principle to find high-frequency harmonic Maxwell equation solutions for many complicated objects. The Uniform Theory of Diffraction (UTD) was proposed in 1974 to complement GTD in situations where GTD was invalid [13]. The GTD/UTD framework has become the practical framework in narrowband radio propagation [18,20] and will be also the foundation of our proposed framework. Thirdly the impulse response waveform of the finite three-dimensional target was approximated by a linear system theory in UWB radar in 1965 [9]. The chief value of the impulse response concept was to sum up the radar signature of an object in a unique manner which adds insight into the complex relationship between the target shape and radar reflectivity. A linear system is fully characterized by its impulse response. Thus in general impulse response fully describes the properties of the radar target for any frequency. Finally this concept of impulse response and scattering center widely used for a UWB radar was borrowed in 1995 [43], to describe the impulse response of a generalized multipath path in UWB wireless communications [22-26]. The resultant generalized model [32-43] extended the widely used Turin’s model [21] that was first suggested in 1958. This model already finds some applications [44-51]. This section is aimed at a tutorial logical development of a unified theory based on this generalized model (also called physics-based generalized multipath model). The starting point of this theory will be Maxwell’s equations. We will address all the aspects that are important from the theory and engineering point of view.

2.2.2 UWB Pulse Propagation Physics and Mechanisms

The starting point of our unified theory that takes into account pulse distortion is the Maxwell’s equations. Maxwell’s equations are linear. The UWB propagation medium is known to be also linear. Therefore the linear system theory and Maxwell’s equations are valid everywhere. We will approach our problem using exact solutions and high-frequency GTD/UTD approximations. Although we will give exact solutions for two special cases (half-plane and wedge), we will focus on developing the GTD/UTD framework that is practical in UWB communications.

GTD/UTD may be viewed as an asymptotic (wavelength $\lambda \to 0$ or $\omega \to \infty$) theory of the solutions of Maxwell’s equations. The algorithms of GTD/UTD are rather simple and lead to explicit expressions for the desired quantities. The short-wave (or high-frequency) behavior of the known exact solutions (diffraction by a wedge, cylinder, and sphere) reveals that the laws of geometric optics break down only in narrow transition regions where diffraction fields are described by GTD/UTD coefficients. Fortunately the narrow transition regions are local, i.e., the diffracted fields only depend on the local properties of the incident waves and bodies in these regions. Fock’s localization principle (1946) [17,18] is a cornerstone of our proposed framework. With Fock’s principle, we postulate that the complex structures can be decomposed into separated scattering centers where each scattering center is
described by an impulse response \( h_i(\tau) \) of the \( l \)-th path. When a ray is distorted in pulse shape, we call this ray a “generalized path” to differentiate it from the conventional one used in Turin’s model [21]. Therefore, all ray fields can be described by the ordinary geometric optics (GO) and GTD (UTD). All UWB signals’ energy is transferred along these ray tubes. Let us consider an example to illustrate how a UWB pulse travels along a generalized path. The amplitude of a diffracted ray \( (u_d) \) is proportional to the amplitude of the inducing primary ray \( (u_i) \) at the point of incidence \( (r_0) \). Mathematically the diffraction coefficient \( D(j\omega; t, t_d) \) connects \( u_i \) with \( u_d \) [17]

\[
u_d = (1/jJ) \cdot e^{iks} \cdot D(j\omega; t, t_d)u_i
\]  

(2.1)

where \( J \) is the Jacobian of the transition to ray co-ordinates, \( s \) the eikonal defining the phase structure of the field, \( t \) and \( t_d \) unit vectors of the incident and diffracted rays, \( k \) the wavenumber, and \( \omega \) the angular frequency. In Eq. (2.1) the coefficient of diffraction depends on the local geometry of the body in the vicinity of the incident ray in the case of corners and edges. Below we will use half-plane and wedge as two examples to show the method of obtaining \( D(j\omega; t, t_d) \) using GTD/UTD. Other complicated geometric structures in engineering applications can be also worked out. For these structures, often we need trace all the significant rays to capture the major energy. Before we show the examples, let us consider the impact of the \( D(\omega; t, t_d) \) on incident pulse waveform.

**UWB Pulse Waveform Distortion**

We can model the localized diffraction phenomenon as a linear system. The transfer function of this linear system is determined by the local diffraction coefficient \( H(j\omega) = D(j\omega; t, t_d) \) located at the point of emanation of the ray \( (r_0) \) on the edge or apex. When a pulse travel through a locally located diffraction point, the pulse is modeled as passing through an equivalent linear system of \( H(j\omega) \). This point acts as a linear filter. In reality this mathematical point is a localized region (called scattering center) [15]. To apply the linear system theory, our observation must be far enough from the scattering center, for the approximation conditions of the high-frequency GTD/UTD to be valid. For a given pulse of incident waveform \( p(t) \), the pulse waveform right after \( r(t) = p(t) \otimes h(t) \) where \( h(t) \) is the inverse Laplace transform of \( H(j\omega) \) [12]. Along the tube of this diffracted ray, the pulse waveform is in the form of \( r(\tau - r_0) = r_0(\tau) \star \delta(\tau - r_0) \) where the delay is \( \tau_0 = s/c \) with \( c \) being the speed of light. In other words, it follows that

\[
r(t) = p(t) \otimes h(t) \otimes \delta(t - r_0)
\]  

(2.2)

The distortionless condition of a linear system is the condition of \( h(t) = \delta(t) \). This condition is valid for Turin’s model but generally invalid for the physics-based model. Turin’s model only takes into account the effect of delays through \( \delta(\tau - r_0) \), as included in (2.2). For example, for diffraction by a perfectly conducting (PEC) half-plane, it follows from GTD that \( h(\tau) = A_0/\sqrt{\tau}U(\tau) \) [16,19] where \( A_0 \) is real and \( U(x) \) a unit function. The mathematical property of this diffraction phenomenon is very far-reaching. The role of the impulse response of the PEC half-plane is the semi-integral of the waveform of the incident primary ray field, with a notation of \( (d/d\tau)^{-1/2} \) or \( (j\omega)^{-1/2} \). Fractional calculus has proven to be a very good tool to handle these problems [33,34]. See subsection 2.2.7 for details.

**Pulse Diffraction by a Perfectly Conducting Half-Plane**

As an example let us consider the canonic problem to illustrate the mechanisms. This problem, first solved by Sommerfeld, is both theoretically and practically important. For
monochromatic incident field $u = A_0 \exp(j\omega t)$, the complex-valued amplitude $u$ of the scattered field at the distance $r$ is given by [18]
\[
    u(j\omega) = u_1 + u_2 = A_0 e^{jkr \cos(\varphi - \varphi_0)} F(\sqrt{2kr} \cos(\varphi - \varphi_0)) \pm A_0 e^{jkr \cos(\varphi + \varphi_0)} F(\sqrt{2kr} \cos(\varphi + \varphi_0))
\]  

where $F(x) = \frac{e^{-x^2}}{\sqrt{\pi}} \int_x^\infty e^{-\mu^2} d\mu$ and the plus and minus signs denote the H and E polarization incident field. $\varphi_0$ and $\varphi$ are, respectively, the incident and observation angles. $k = \omega / c$ is the wave number. The Fresnel function $F(x)$ can be expressed in terms of the error function $\text{erfc}(x)$ as
\[
    F(x) = \frac{1}{2} \text{erfc}(e^{-x^2/4})
\]

where $\text{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-\xi^2} d\xi$. Using the Laplace pair of an error function, we can derive the closed form of the impulse response defined by $h(\tau) \triangleq L^{-1}\{u(j\omega)/u_i\}$
\[
    h_{\text{half-plane}}(\tau) = \frac{\sqrt{2r/c}}{2\pi} \left[ \frac{\cos(\varphi - \varphi_0)}{\tau + \frac{r}{c} \cos(\varphi - \varphi_0)} \pm \frac{\cos(\varphi + \varphi_0)}{\tau + \frac{r}{c} \cos(\varphi + \varphi_0)} \right] \frac{1}{\sqrt{\tau - r/c}} U(\tau - r/c)
\]  

where $U(t)$ is the unit function of $t$. The transient exact solution is remarkably simple, compared with its time-harmonic frequency domain solution (Eq. 2.3). To compare our results with others, consider the normal incidence where $\varphi = 2\pi - \theta$ and $\varphi_0 = \pi$. Eq. (2.4) reduces to
\[
    h_{\text{normal}}(\tau) = \frac{\sqrt{2r} \sin \theta}{2 \pi c r - \tau \cos \theta} \frac{1}{\sqrt{\tau - r/c}},
\]

which is identical to the derivative of the unit step response results obtained by [7]. Reference [11] also obtained similar expressions.

Eq. (2.4) has two singularities: (1) when $\tau + \frac{r}{c} \cos(\varphi - \varphi_0)$ and $\tau + \frac{r}{c} \cos(\varphi + \varphi_0)$ approach zeros; (2) when $\tau$ approaches $r/c$. We only consider the second case where the wave front $c\tau - r$ assumes a singularity of $(1/\sqrt{\tau})U(\tau)$ if the singularity of case 1 is absent. We only have one scatter center occurring at the edge of the half-plane. If a pulse $p(t)$ propagates through this scattering center, the pulse waveform will be distorted since the impulse response of the half-plane is not a Dirac function $\delta(\tau)$.

In Keller’s GTD model diffraction problem for a half-plane edge [16,19], the monochromatic diffraction wave is given as $u_{d1} = \frac{1}{\sqrt{r}} e^{-jkr} D(\varphi, \varphi_0, k) u_i$ where
\[
    D_{s,h}(\varphi, \varphi_0, k) = \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k}} \left( \frac{1}{\cos[(\varphi - \varphi_0)/2]} \pm \frac{1}{\cos[(\varphi + \varphi_0)/2]} \right)
\]  

$D_{s,h}(\varphi, \varphi_0, k)$ is a diffraction coefficient obtained through the first-order asymptotic approximation of the exact solution of Eq. (2.3). This first-order solution breaks down within the transition zone, known as semi-shadow, which is defined by the region $q \leq q_0$ where $q$ is defined as $q = \sqrt{2kr} \cos \frac{\varphi_0}{2}$ and $q_0$ is the minimum value of $q$ such that the approximation Eq. (2.5) is satisfactory within $q \geq q_0$. With
the aid of a Laplace pair, from Eq. (2.5) the time domain diffraction coefficient predicted by Keller’s GTD is derived as
\[ h_{s,b}(\tau) = -\frac{\sqrt{c}}{2\pi\sqrt{2}} \left( \frac{1}{\cos[(\phi - \phi_b)/2]} \pm \frac{1}{\cos[(\phi + \phi_b)/2]} \right) \frac{1}{\sqrt{\tau - r/c}} U(\tau - r/c) \] (2.6)

**Pulse Diffraction by a Perfectly Conducting Wedge**

As another example we consider the wedge. The exact transient solution to a perfectly conducting wedge is available [6] as follows (for \( \Omega > \pi \) and \( \phi' < \Omega - \pi \))
\[ h_{\text{wedge}}(r, r'; \tau) = \frac{\delta(\tau - |r - r'|/c)}{4\pi |r - r'|} U(\pi - |\phi - \phi'|) + \frac{\delta(\tau - |r - r'|/c)}{4\pi |r - r'|} U(\pi - |\phi - \phi'|) \]
\[ -\frac{c}{4\pi^2} \frac{Q(\phi, \phi', \beta) + Q(\phi, -\phi', \beta)}{\mu \rho' \sinh(\beta)} U(c\tau - D_0) \] (2.7a)

where
\[ Q(\phi, \phi', \beta) = -\frac{\pi}{2\Omega} \left\{ \frac{\sin[(\pi / \Omega)(\phi - \phi' - \pi)]}{\cosh(\pi \beta / \Omega) - \cos[(\pi / \Omega)(\phi - \phi' - \pi)]} \right\} \]
\[ -\frac{\sin[(\pi / \Omega)(\phi - \phi' + \pi)]}{\cosh(\pi \beta / \Omega) - \cos[(\pi / \Omega)(\phi - \phi' + \pi)]} \]
\( \Omega \) is the angle of the wedge. Polar coordinates \((\rho, \phi, z)\) and \((\rho', \phi', z')\) are used for observation point and source point, respectively. (2.7a) assumes that Neumann boundary condition on the wedge faces. Parameters \( \beta, T, D_0 \) are defined by
\[ \beta = \cosh^{-1}[(c^2 T^2 - \rho^2 - \rho'^2)(2 \rho \rho')^{-1}] \]
\[ c T = [c^2 \tau^2 - (z - z')^2]^{1/2} \]
\[ D_0 = [(\rho + \rho')^2 + (z - z')^2]^{1/2} \]

There is only scattering center in this case. The first two terms represent the geometric optic ray fields. The third term represents the diffracted field caused by an impulse plane wave. Obviously (2.7a) will predict some pulse waveform distortion if a pulse signal is diffracted by a perfectly conducting wedge. The transient responses given in (2.4) and (2.7a) are valid at all observation times, thereby providing an insight into the field behavior at observation times immediately (early-time) and long (late-time) after the passage of a wavefront. The time domain GTD (Keller’s) coefficient has been derived as follows [16,19]
\[ h_{s,b}(\tau) = A \frac{1}{\sqrt{\tau - r/c}} U(\tau - r/c) \] (2.7b)

where
\[ A = \frac{\sqrt{c}}{\pi \sqrt{2}} \frac{\sin \pi / n}{n \pi} \left( \frac{1}{\cos \pi / n - \cos[(\phi - \phi_0)/n]} + \frac{1}{\cos \pi / n - \cos[(\phi + \phi_0)/2]} \right). \]

The two faces of wedge is \( \phi = 0 \) and \( \phi = n\pi \). The impulse response obtained from UTD has been derived as [16,19]
\[ h_{s,b}(\tau) = \sum_{m=1}^{4} A_m \frac{1}{\sqrt{\pi \tau c (\tau + X_m / c)}} U(\tau) \quad \tau = t - r/c \] (2.7c)

where \( A_m \) and \( X_m \) are time-independent parameters. For practical UWB pulses we find that the methods (GTD, UTD, and Exact) in (2.7) give us calculated pulse waveforms that are not differentiable [37]. (2.7b) and (2.7c) can readily shown to have a singularity of type \( \tau^{-1/2} U(\tau) \) in the wavefront \( \tau = t - r/c \). More generalized singularities will be given in (2.11) and (2.12) as well as (2.19) and (2.20).
2.2.3 Early-Time Response and Asymptotic High-Frequency Time-Harmonic Solutions

Let us investigate the relationship between the early-time response and asymptotic high-frequency time-harmonic solutions, due to its fundamental nature in our unified theory. A basic difficulty of the GTD/UTD based Laplace approach is the validity of the GTD/UTD based high-frequency approximation of the exact solutions of each individual scattering center [12,16,19]. The communications medium can be regarded as the composite interactions of separated scattering centers [39,49,50]. Due to high-resolution capability of a UWB pulse signal, these scattering centers are well resolved in the time domain. This is one of the most fundamental features of a UWB system from a signal analysis point of view. The property of each scattering center can be treated as different (or composite) mechanisms corresponding to reflection, refraction, and diffraction [15]. Under the GTD/UTD framework, each mechanism is fully described by GTD/UTD coefficients. The accuracy of these time domain GTD/UTD coefficients is not well established. Can we use the time domain GTD/UTD coefficients to model the modern UWB communications/sensing systems? Fortunately we have already made a decisive step [37] to answer this. Compared with the exact solutions of wedge and half plane, the time domain GTD/UTD coefficients are shown to be sufficiently accurate for the modern UWB communications/sensing system modeling. This has paved the way to a more systematic use of the time domain GTD/UTD coefficients as our building blocks in UWB signals modeling. These closed form solutions based on UTD/GTD provide big advantages over the conventional methodology.

The mathematical foundation of the GTD/UTD is the Kline-Luneberg expansion of the field components. The main result of this subsection is to tell us how to obtain the asymptotic series representation of the steady or time-harmonic solution of Maxwell’s equations if we know the behavior of the corresponding pulse solution in the neighborhood of its singularities [7]. We will give the form of asymptotic series that is valid in terms of the singularities of the pulse solution. The asymptotic series is determined only by the singularities, singularities of the pulse solutions. The mathematical result of this expansion is intimately related to an Abelian theorem (Watson’ Lemma) in Laplace transform theory [31]. This theorem states [7] that if the original function $h(t)$ has the convergent or asymptotic form

$$h(t) \approx \sum_{n=0}^{\infty} C_n t^{n} \quad \text{for } t \text{ near } 0^+ \text{ and } n > -1$$

and $h(t) < 0$ for $t<0$ (causal), then the image function $H(s)$ of $h(t)$ has the asymptotic form

$$H(s) \approx \sum_{n=0}^{\infty} C_n \frac{\Gamma(1+n)}{s^n} \quad \text{as } s \to \infty$$

In this theorem $s = j\omega + \sigma$ may be complex where $\sigma$ is greater than zero. This theorem connects the early-time response (the initial transient fields on and near the wavefront) and the high-frequency time-harmonic solutions. Let us assume the field near the wavefront $t = r/c$ behaves as $h(r,t) \approx A(t-r/c)^\beta$, where $A$ is time independent and $\beta > -1$. This is the case for Eqs. (2.4) and (2.6). The asymptotic behavior of the field behaves as

$$H(j\omega) \approx \exp(jkr) A \frac{\Gamma(\beta+1)}{2\pi(j\omega)^{\beta+1}}$$

This is true for Eq. (2.5). Recall that GTD/UTD solutions are asymptotic solutions of the model problem solutions that are exact. The field behavior near $t = r/c$ is related to the time-harmonic behavior for $\omega \to \infty$.

Eq. (2.10) is inadequate to describe the phenomena at times $t \approx r/c$ corresponding to arrival of the wavefront. As described above, the wavefront fields are synthesized by the high-frequency time-harmonic wave constitutes. The asymptotic expansions of the time-harmonic fields (such as obtained by GTD/UTD) can be employed to develop corresponding expansions of the transient fields for $t \approx r/c$ (early time response), improving the leading term expressions of (2.10). A generalized version of the time-harmonic field is given by [5]
then, from (2.8) and (2.9), the transient field near $t = \psi(r)/c$ is

$$h(\tau) \approx A'(r)\delta(\tau) + \tau^B \sum_{n=0}^{\infty} A_n(r)\tau^n, \quad \tau = t - \Psi(r)/c \geq 0$$

(2.12)
right-angle (90 degree) wedge, where x is the distance from the edge (scattering center). At high frequencies, the field tends to be localized (Fock’s principle of locality). The energy tends to be localized in the scattering center (a singularity). If we have a UWB pulse of duration of T nanoseconds (ns), the condition to use the time-domain GTD to describe the pulse response of the scattering center is that the UWB pulse duration T must be smaller than the useful period of \( \Delta t \sim \frac{10}{c} \), i.e., \( T < \Delta t \sim 10x/c \). For \( T = 0.1-1\,\text{ns} \) (1-10GHz bandwidth), \( x > cT/10 = 3T = 0.3-3\,\text{cm} \), where \( c=30\,\text{cm/ns} \) (speed of light). In other words, the observation point must be 0.3-3 cm away from the scattering center.

### 2.2.4 Impulse Response for Electrically Large Objects

Before generalizing the above results in a heuristic manner to model the UWB channel (Subsection 2.2.5), we must introduce the method in the literature for modeling large radar objects. The same method will be still valid for UWB pulse propagation in communications environment. Physically and mathematically the two problems are equivalent. Radar interrogation (per-path pulse response modeling) is essentially a transient process [9]. In order to simplify the calculation of radar reflectivity (per-path pulse response), one usually assumes that values obtained for a monochromatic source (time-harmonic solutions) are representative of the target (per-path pulse response) and can be used with little or no modification, to predict the return for a pulsed signal with the same carrier frequency. This assumption cannot be made for ultrashort pulses or any other UWB signaling waveform. To obtain a target reflectivity model (per-path pulse response model) applicable to all types of signal, time-domain concepts suggest use of an impulse response waveform to characterize each radar target (per-path pulse response).

The first such attempt to estimate an impulse response waveform in the radar scattering context was perhaps that of Kennaugh and Cosgriff (1958). Using extremely simple approximations for high- and low-frequency backscatter from conducting sphere and spheroids, estimates of the impulse response waveform were obtained. The mathematical physical tools to attack the problems involving finite three-dimensional targets are described in Section 2.2.2. As suggested in Eq. (2.1), a linear system can be used to describe the object. We will follow [9] (1965) in the following derivations of this subsection.

![Figure 2.1 Coordinates of impulse response definitions](image)

Figure 2.1 Coordinates of impulse response definitions

To apply linear system analysis to scattering analysis problems [39,49,50], we first define input and output quantities as electrical field intensities at a pair of points in space and in fixed directions. As shown in Fig. 2.1, the input \( x(t) \) is the x-directed electric field intensities at the origin produced by a plane wave \( E' = \dot{x}(t-z/c) \). If, now, this plane wave is incident upon an arbitrary scattering object (per-path obstacles), a scattered electromagnetic field \( E' \) is produced. We may select an arbitrary component of this vector field at any point \( P(r) \) to be the output of \( y(t) \). Thus, restricting the direction and polarization of the incident plane wave, the relationship between the \( x(t) \) and \( y(t) \) is that of a one-dimensional linear system. If we assume that the scatterer does not move, so that the system is time-invariant. The impulse response waveform \( h_0(t) \) is that obtained when the incident plane wave \( x(t) \)
is impulsive, i.e., \( x(t) = \delta(t) \), the Dirac delta-function. Several features of the impulse response waveform can be inferred immediately if the scattering body is of finite size:

1. \( h_0(t) \) is identically zero for \( t < 0 \) if the scatterer lies to the right of the xy plane, as in Fig. 2.1. The linear system must be causal.
2. \( h_0(t) \) decays exponentially for large values of \( \tau = t - r/c \), where \( r \) is the distance to the observation point from the origin, as in Fig. 2.1.
3. When \( r \gg L \), where \( L \) is the maximum dimension of the scattering object, \( h_0(t) \) varies with \( r \) in a given scattering direction as \( r^{-1} f(t - r/c) \) for transverse components of the scattered field.

**Properties of Impulse Response Waveform \( h(t) \)**

The normalized impulse response \( h(t) \) and the transfer function \( H(j\omega) \) are Laplace transform pair (a causal linear system). Here \( H(j\omega) \) is related to a transverse component \( E^r(t - r/c) \) of the scattering field at large \( r \) by

\[
\text{Re}\left\{ H(j\omega)e^{j\omega(t-r/c)} \right\} = (2r/c)E^r(t - r/c)
\]

(2.13)

the incident plane wave being given by \( E^i = \hat{x}\cos\omega(t - z/c) \). Using \( s \) notation,

\[
H(s) = \int h(t)e^{-st}dt, \quad H(s) = L\{h(t)\}, \quad h(t) = L^{-1}\{H(s)\}
\]

(2.14)

In practice we usually first obtain \( H(j\omega) \) using the GTD/UTD asymptotic solutions, then we can replace \( sj \) in \( H(j\omega) \) to obtain \( H(s) \). Secondly we apply the Laplace transform pair to obtain the \( h(t) \). Not all the functions have their inverse Laplace transform in closed forms. Due to the essential property of casualty, a Laplace transform rather than a Fourier transform must be used, in order to guarantee that the so obtained impulse response is physically sound. Such obtained impulse response must be a real function. This can serve as sanity check in the analytical analysis.

**High-Frequency Impulse Response Based on Scattering Centers**

Let us present such a model based on the concept of scattering centers. Extensive development of high-frequency scattering theory (developed in subsections 2.2.2 and 2.2.3) traced back at least to 1940s-60s. The basic work was dated back to Sommerfeld (1901) [1,2] and early acoustics [2-4]. Experimental conditions and analytic tools were mature until 1960s, mainly driven by the study of short pulses caused by nuclear explosions. Two categories of work were performed. One was to model the impulse response of simple shapes and composite shapes (finite three-dimensional objects). Another category was to study the radio propagation of pulse in a dispersive medium like plasma at a distance of kilometers. Here we are only interested in methods developed for the first category, especially based on experimental work [10,27].

It is postulated that for a sufficiently short pulse, the solution can be represented as a sequence of pulses scattered from the resolved scattering centers. (Subsection 2.2.2 has discussed the conditions of validity of the time domain GTD to model the resolved scattering centers.) This postulated solution has an approximate equivalent in the frequency domain. The back scattering amplitude at the frequency \( \omega \) will be of the form

\[
H(j\omega) = \sum_n A_n H_n(j\omega)e^{-j2k\tau_n} = \sum_n A_n H_n(j\omega)e^{-j\omega\tau_n}, \quad \tau_n = \frac{L_n}{c}
\]

(2.15)

where \( A_n \) is the scattering amplitude, \( H_n(j\omega) \) is the frequency response for the \( n \)th center, and \( L_n \) is the distance along the ray path from the first to the \( n \)th scattering center. The so-called per-path transfer function (heuristically defined in 2.2.5), \( H_n(j\omega) \), plays the central role in characterizing the pulse...
The per-path delay, $\tau_n = \frac{2L_n}{c}$, describes the intra-path delay that connects the localized responses. When an inverse Laplace transform is applied in (2.15) (see Eqs. (2.8) and (2.9)), the early-time approximation to the impulse response is obtained as

$$h(\tau) = \sum_n A_n h_n(\tau) \ast \delta(\tau - \tau_n)$$  \hspace{1cm} (2.16)

Both $H_n(j\omega)$ and $h_n(\tau)$ have a unit energy for convenience. In general (2.15) and (2.16) are connected by the Laplace transform. We assume that $H_n(j\omega)$ is centered about $f_0$ with a normal bandwidth $B$. If $H_n(j\omega)$ is slowly varying for $-B/2 \leq f \leq B/2 + f$. The term $e^{-j2\pi f_0}$ varies rapidly in the same band. For a special case, a simple relation can be obtained as

$$h(\tau) = \sum_n A_n H_n(\omega_0) p(\tau - \frac{2L_n}{c})$$  \hspace{1cm} (2.17)

where $p(t)$ is the incident pulse. Thus the $n$th scattering center returns a pulse of amplitude $A_n H_n(\omega_0)$, delayed by $\tau_n = \frac{2L_n}{c}$ from the first term. From a physical point of view, (2.16) and (2.18) imply that the dimensions of the scattering centers are of the order one wavelength or less. Thus, if the incident pulse contains more than several wavelengths (at the center frequency), the distortion of the scattered pulses is negligible. Refinements of (2.18) could be included by accounting for the frequency dependency of $H_n(j\omega)$ near the center frequency, as in (2.16). It should be noted that this representation in (2.16) and (2.17) may also include terms due to multiple scattering.

For high-frequencies, one may approximate $H(s)$ by an expansion called Luneberg-Kline series in negative powers of $s$, (as discussed in subsection 2.2.3 and used in subsection 2.2.5). Individual terms in such an expansion are related to impulse, step, ramp, and low-order discontinuities in $h(t)$. Thus, if the asymptotic expansion of $H(s)$ contains a term $A_n s^{-n} e^{-\sigma t}$, the impulse response waveform undergoes a discontinuous jump of magnitude $A_n$ in its $(n-1)$th order derivative at $t = \tau$. Terms of the form $A_n e^{-\sigma t}$ in the asymptotic expansions are contributions predicted by geometric optics, giving impulses in $h(t)$ at $t = \tau$, of magnitude $A_n$. It is clear that low-frequency approximation seres to determine waveform shape and size, whereas high-frequency information relates to fine structure and detail in the waveform. Both are thus interpreted in a single concept that serves to predict the scattering behavior in the intermediate or resonance range of frequencies. The intuitive hypothesis that ramp response is most closely associated with the volume and size of a scatterer suggests that, as our knowledge of response waveforms for special objects increases, we shall ultimately be able to sketch, by inspection, a reasonable accurate estimate of the ramp response of any target (per-path response).

2.2.5 Generalized Multipath Model Based on Per-Path Pulse Waveform

The methods and mathematical physical results are historically developed for UWB radar (section 2.2.4), but can be generalized to a UWB channel if we treat the objects encountered by tracing a path in multipath channel modeling. This will be the philosophy adopted in this subsection. Admittedly, the state of the art in predicting or approximating impulse response waveforms for per-path is rather primitive. It has been applied to perfectly conducting objects of simple shape [33-43]. With a combination of theoretical and experimental techniques, the associated waveforms can be determined for other, more complex, shapes. This work is being carried out by our research group in the laboratory at TTU. No technique or concept can make intrinsically difficult task simple, and the long history of channel modeling investigations indicate that this is, indeed, a difficult task. Perhaps the chief value of the per-path impulse response concept is that it sums up the signal signature of a path (like the radar
signature of an object) *in a unique manner* which adds insight into the complex relationship between each individual path response and the propagation environment.

![Diagram](image)

Fig. 2.2 The Structure of the Generalized Multipath Model

Based on the GTD framework and principle of locality [8], we postulate that the total response is modeled by the sum of the impulse responses of local scattering centers [33-43]:

\[
h(\tau, \theta, \phi) = \sum_{n=1}^{N_1} a_n(\theta, \phi) \delta(\tau - \tau_n) + \sum_{n=1}^{N_2} A_n(\theta, \phi) h_n(\tau, \theta, \phi) * \delta(\tau - \tau_n)
\]

(2.18)

where \(N_1\) and \(N_2\) represent the number of rays without per-path pulse distortion and rays with per-path pulse distortion, respectively. The first term is of the form identical to Turin’s model. The second term is more generalized than the first term that is a special form of the second term when \(h_n(\tau) = \delta(\tau)\).

\(h_n(\tau)\) has a unit energy. When an arbitrary pulse, \(p(t)\), passes through the channel defined by Eq. (2.18), some pulses will be undistorted and some pulses will be distorted. When the duration of this pulse, \(T_p\), is shorter than any interpath interval of Eq. (2.18), i.e., \(T_p < |\tau_i - \tau_j|\) for \(i \neq j\), all pulses will be resolvable in time. The \(h_n(\tau)\) can be obtained through exact, experimental, numerical or/and asymptotic methods. As an example, let us take a look at the diffracted rays. The geometric optics rays can be treated as a special form of a generalized diffracted ray [7]. If in the neighborhood of the singularity \(\tau = \tau_\alpha\), any field component has the behavior

\[
h_n(\tau) = \begin{cases} 
    \xi(\tau - \tau_\alpha) \sum_{n=0}^{\infty} C_n (\tau - \tau_\alpha)^n, & \tau < \tau_\alpha \\
    \eta(\tau - \tau_\alpha) \sum_{n=0}^{\infty} D_n (\tau - \tau_\alpha)^n, & \tau > \tau_\alpha 
\end{cases}
\]

(2.19)

where \(\xi(\tau)\) and \(\eta(\tau)\) are rather general, then the corresponding asymptotic frequency response has the form [7]

\[\text{Note the properties of Dirac’s function: } \delta(\tau) \otimes \delta(\tau) = \delta(\tau) \text{ and } \delta(\tau) \otimes \delta(\tau - \tau_n) = \delta(\tau - \tau_n).\]
\[ H_n(j\omega) \approx e^{-jk\Psi} \sum_{m=0}^{\infty} \left\{ \frac{D_m}{m!} \left( j\omega \right)^m \int_0^{\infty} \eta(\frac{t}{j\omega}) t^m e^{-t} dt - \frac{C_m}{m!} \left( -j\omega \right)^m \int_0^{\infty} \xi(\frac{t}{-j\omega}) t^m e^{-t} dt \right\} \]

(2.20)

where \( \Psi(x,y,z) \) determines the wavefront at the \((x,y,z)\) in question, i.e., \( \Psi(x,y,z) - ct = 0 \) is the discontinuity hypersurface on which \( h_n(\tau) \) has a finite jump. As shown in subsection 2.2.3, the behavior of the impulse response solution in the neighborhood of its singularities does determine the asymptotic series for the corresponding time-harmonic problem. Further we learn the form of the asymptotic series in terms of the form of the singularity of the impulse response solution. The function of (2.19) can be infinite at the singularities. The results of (2.19) and (2.20) extend a known result in Laplace transform theory—the Abelian theorem (Eqs. (2.8) and (2.9)).

Eq. (2.20) is a special form of Eq. (2.18). Eqs. (2.18) and (2.20) are insufficiently generic to represent all the signals in the set of the actual physical field signals that satisfy Maxwell’s equations. But they are generic enough to describe almost all the practical series obtained by GTD/GTD and other high-frequency asymptotic techniques. For a large category of physical signals, both \( \xi(\tau) \) and \( \eta(\tau) \) are of a form \( \tau^{\frac{1}{2}} \), the asymptotic series contains fractional powers of \( 1/(\omega) \), e.g., \( 1/(j\omega)^{m+\frac{1}{2}} \) where \( m \) is an integer. For diffracted signals \( \xi(\tau) \) and \( \eta(\tau) \) can take a more general form of \( \tau \left( \log \tau \right)^m \), \( \exp(-1/\tau) \), and \( \tau^\alpha \) where \( \alpha \) is real [7]. \( \xi(\tau) \) and \( \eta(\tau) \) may also have non-algebraic singularities provided that these functions are continuous at \( t = 0 + \). For \( \omega \to \infty \) each term of the series vanishes. This is as expected, for there is no classical diffracted geometric optics field [7]. The leading term for large \( \omega \) may well serve as the geometric optics diffracted field. For a special case, using a Laplace transform pair, we obtain

\[ h_n(\tau) = \begin{cases} \frac{1}{\sqrt{\tau}} \sum_{m=0}^{\infty} \frac{D_m}{m!} (\tau - \tau_0)^m, & \tau > \tau_0 \\ 0, & \tau < \tau_0 \end{cases} \]

\[ H_n(\omega) = \sum_{m=0}^{\infty} \frac{D_m}{(j\omega)^{m+\frac{1}{2}}} \Gamma(m + \frac{3}{2}) \]  

(2.21)

The primary term in Eq. (2.21) is sufficient in practice (e.g., GTD/UTD series). Now consider the physics-based generalized multipath model of a form in the following

\[ H(\omega) = \sum_{n=1}^{N} A_n(j\omega)^{\alpha_n} e^{i\omega \tau_n} \quad h(\tau) = \sum_{n=1}^{N} \frac{A_n}{\Gamma(-\alpha_n)} \tau^{-(1+\alpha_n)} U(\tau) \otimes \delta(\tau - \tau_n) \]  

(2.22)

where \( \alpha_n \) is a real value, \( H_n(j\omega) = (j\omega)^{\alpha_n} \) and \( h_n(\tau) = \tau^{-(1+\alpha_n)} U(\tau) \). \( U(x) \) is Heaviside’s unit function. This model is asymptotically valid for incident, reflected, singly diffracted and multiple reflected/diffracted ray path field. The motivation of using the function in Eq. (2.22) is its simplicity and feasibility for a large category of problems. This form can allow us to use the powerful mathematical tool of fractional calculus [29,30] to conveniently describe our problems. Eq. (2.18) gives a formal generic mathematical model for \( h(\tau) \). A large category of problems can be described by per-path impulse response \( h_n(\tau) \) given by Eq. (2.20). In (2.22), we can model \( \alpha_n \) as the random variables that can be experimentally measured. The amplitudes and delays can be statistically described similarly to the Turin’s model.
Fig. 2.3 Pulse waveform distortion as a function of \( \alpha \) for one generalized multipath.

As an example, in Fig. 2.3, we obtain the pulse waveform distortion as a function of \( \alpha \) for one generalized multipath defined by its impulse response \( \tau^\alpha U(\tau) \). The incident pulse waveform is the second order derivative of a Gaussian function (corresponding to the curve of \( \alpha = 0 \)). Most time a matched filter is required for an optimum reception (see next subsection), so the new changed waveform after the incident pulse passing through the sole generalized multipath is needed and calculated in Fig. 2.3. For a pulse position modulation, a template pulse consisting of the incident pulse and its negative delayed version is used for detection. This template pulse is only proper when the pulse waveform is not changed by the channel. When a pulse is distorted by passing through a generalized multipath, then the pulse waveform is changed to a new waveform. Thus a new template formed by the new changed waveform and its delayed version should be used [36,38].

### 2.2.6 Impact of Propagation Processes on Optimum Receiver Structures

We will incorporate the per-path impulse response into the optimal receiver when inter-symbol interference (ISI) is present. Several different modulations have their transmitted signals in the same form [32] as

\[
s(t) = \sum_{n=0}^{\infty} a_n x(t - nT),
\]

where \( a_n \) represents the discrete information symbol sequence with symbol duration of \( T \), and \( x(t) \) is a pulse that is bandlimited. The channel impulse response \( h(\tau) \) is illustrated in Fig. 2.2 for the generalized multipath (Subsection 2.2.5). The receiver signals are represented as

\[
r(t) = \sum_{n=0}^{\infty} a_n y(t - nT) + n(t)
\]

where \( n(t) \) is AWGN. The received symbol signal waveform is \( y(t) = h(t) \otimes x(t) \), consisting of a sequence of distorted pulses described by \( h(t) \) defined by (2.18). It has been shown that the optimum receiver structure is illustrated in Fig. 2.4. The received signal \( r(t) \) first passes the matched filter followed by a sampler of sampling rate of \( 1/T \). The sampled sequence is further processed by a Maximum Likelihood Sequential Estimation (MLSE) detector. This receiver structure is optimal in terms of minimizing the probability of transmission error in detecting the information sequence. If we
accept the condition that $y(t)$ is finite in the UWB receiver, this structure can immediately be applied to our problem. The output of the matched filter is expressed as

$$q(t) = \sum_{n=0}^{\infty} a_n R_{yy}(t-nT) + v(t)$$

(2.24)

where

$$R_{yy}(t) = y(t) \otimes y^*(-t) = R_{xx}(t) \otimes h(t) \otimes h^*(-t)$$

is the autocorrelation of $y(t)$ and $v(t)$ is the response of the matched filter to AWGN noise $n(t)$. $R_{xx}(t)$ is the autocorrelation of $x(t)$. Pulse distortion will affect the optimal energy collection in matched filter. The sampled values at $t=nT$ can be expressed as

$$q_k = a_k + \sum_{n=0, n \neq k}^{\infty} a_n R_{k-n} + v_k$$

(2.25)

where $a_k$ term represents the expected information symbol of the $k$-th sampling period, the 2nd term is the ISI, and $v_k$ is the additive Gaussian variable at the $k$-th sampling point. The sequence of $q_k$ will be further processed by MSLE (Viterbi algorithm) before being sent to decision circuits. After pulse distortion is taken into account in the ISI through (2.24) and (2.25), all the rest of analysis follows the conventional case [32] when Turin’s model rather than (2.18) is used for $h(t)$.

Fig. 2.4 Optimal Receiver Structure in Presence of ISI in a AWGN Channel

The suboptimum implementation of the matched filter is the RAKE receiver structure. Suboptimum linear equalizers such as zero-forcing, decision feedback, adaptive equalization can be used to replace the optimum MSLE. Since the per-path pulse waveform has been included in the channel impulse response (illustrated in Fig. 2.2), the resultant matched filter is sometimes called “Generalized RAKE” receiver structure [33], which is a generalization of Altes’ receiver structure [14].

2.2.7 Representation of Pulse Distortion Using Fractional Calculus

Sommerfeld (1901) was the first to analyze the diffraction of pulses. Following Sommerfeld, Lamb (1910) [2] and Friedlander (1946) [3] analytically and numerically studied the pulse distortion caused by diffraction at a half-plane for sound pulses. Lamb’s work was also valid for electromagnetic waves. Friedlander transformed Lamb’s result into Abel’s integral equation which is the classical result in fractional calculus. We can readily represent their formulation in terms of the Riemann-Liouville fractional differintegral. These earliest two papers [2,3] are the starting point for us to apply fractional calculus in UWB communications. (In electromagnetics, Engheta was the first to use fractional calculus [28].) The Riemann-Liouville fractional differintegral $(d/dt)^\nu$ can be defined as

$$(d/dt)^\nu f(t) \equiv _D^0 D_t^\nu f(t) \equiv \left[ \frac{1}{\Gamma(-\nu)} t^{1+\nu} U(t) \right] \otimes f(t) \quad \nu < 0$$

(2.26)

where $\nu$ is negative and real. For fractional $\nu > 0$, the above definition can be applied with additional step $\ _D^0 D_t^\nu f(t) = (d/dt)^m \ _D^0 D_t^{\nu-m} f(t)$, where $m$ is a positive integer large enough so that $(\nu - m) < 0$. 

13
According to (2.31) we find that $(j\omega)^\nu F(j\omega)$ is the Fourier transform of $\nu D^\nu f(t)$. When $\nu$ assumes a value of integer, $(d/dt)^\nu$ or $\nu D^\nu$ reduces to the ordinary Riemann integral $\nu = n$ where $n$ is an (positive or negative) integer. This new tool provides a powerful means to describe the pulse distortion. For example, for the PEC wedge and half-plane (a special case of wedge), we have $\nu = \alpha = -1/2$. For the generalized multipath signals in (2.22), we have $\alpha = \alpha_n$ for the $n$-th path. So (2.22) can be expressed in a new form as

$$h(\tau) = \sum_{n=1}^{N} A_n (d/d\tau)^{\alpha_n} \delta(\tau) \otimes \delta(\tau - \tau_n) \quad (2.27)$$

where $\alpha_n$ is an arbitrary real number. When a pulse $p(t)$ passes through the channel, the new pulse is

$$r(t) = \sum_{n=1}^{N} A_n (d/dt)^{\alpha_n} p(t) \otimes \delta(t - \tau_n) \quad (2.28)$$

According to (2.27) and (2.28), after a pulse travels through a generalized multipath, its waveform will be changed and represented by the fractional differintegral $(d/dt)^{\alpha}$ of this pulse. The delay of this pulse is similarly modeled as in Turin’s model [21] which ignores the effect of the fractional differintegral $(d/dt)^{\alpha}$ of this pulse on the incident pulse. The fractional differintegral has a lot of desirable properties that greatly simplify the analytical and numerical analysis of the signals in (2.28) [33].

References


45. IEEE 802.15.4a, “Status report of the channel modeling subgroup,” prepared by Chair, Subcommittee on Channel Modeling (A. Molisch), Vancouver, Canada, January 8, 2004.


